An Embedded Link MDP Model with Application to Call Admission Control

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Abstract. In this paper we study the call admission control (CAC) problem for a single link in multi-service loss networks. Each call is described by a reward parameter representing the expected reward for carrying this call. The control objective is to maximize the reward from carried calls. The behavior of the link is modeled as a Markov Decision Process (MDP). The standard link MDP model assumes a Poisson call arrival process and exponentially distributed call holding times. However, some services on the Internet, such as the World Wide Web service, have self-similar call arrival processes and heavy-tailed holding time distributions. In this paper we assume a simple call traffic model consisting of a single call category with general renewal arrivals and exponential holding time distribution. We present an extended link MDP model based on an embedded controlled Markov chain for this traffic model. The numerical results show that the proposed link MDP model provides accurate estimates of the reward loss.

1 Introduction

We consider the problem of Call Admission Control (CAC) on a single link in multi-service loss networks such as ATM and STM networks, and IP networks, provided they are extended with resource reservation capabilities. The objective is to maximize the revenue from carried calls, while meeting constraints on the Quality of Service (QoS) and Grade of Service (GoS) on the packet and call level, respectively.

This paper deals with a particular form of state-dependent CAC on the link level, where the behavior of the link is formulated as Markov Decision Process (MDP) [7, 17]. A MDP is a controlled Markov process, where the set of state transitions from the current Markov state to other Markov states depends on the decision or action taken by the controller in the current state. In the MDP framework, each call is described by a expected reward parameter and the objective is to maximize the reward from carried calls.

A necessary modeling simplification in MDP-based network routing is to decompose the network into a set of links assumed to have independent traffic and reward processes, respectively. When formulating the MDP framework for each link, calls with the same bandwidth requirement are aggregated into a common category.

The first issue of traffic modeling concerns the arrival and service process for each of the G call categories offered to the link. Measurements on real world wide web (WWW), network news (NNTP) and email (SMTP) connection arrivals in the Internet have revealed that the arrival process shows burstiness over many time scales, ranging from seconds to hours. Paxson and Floyd found that actual WAN traffic is consistent with statistical self-similarity for sufficiently large time scales [14]. However, models of WAN connection arrivals include self-similar models [5, 4] and non-self-similar models [9]. In this paper we adopt the model of Anja Feldmann who proposed a certain non-Poisson renewal call arrival process model [9]. The main reasons for adopting a renewal model is its simplicity and the fact this model provides an accurate fit to real TCP connection arrival statistics, see [9]. The particular renewal process has interarrival times that follow a Weibull distribution in contrast to the Poisson process which has exponentially distributed inter-arrival times. For the range of distribution parameters plausible for TCP connection arrivals within WWW sessions, the complementary Weibull distribution decays slower (has a more heavy tail) than the standard exponential distribution. Measurements have also shown that the complementary distribution of holding times of TCP connections within WWW and file transfer (FTP) sessions decays slower than exponentially [4, 5, 14]. In this paper we limit ourselves to the exponential holding time distribution since this case is easier to handle than the case with heavy-tailed holding time distribution.

The second issue of traffic modeling concerns the superposition of per-category renewal arrival and service processes. Cherry presented in [1] an early result that the superposition of two renewal processes is a Markov Renewal Process (MRP), which has an equivalent semi Markov Process (SMP) representation. The SMP is a continuous-time discrete-state Markov process with generally distributed state sojourn times. The MRP may be shown to be equivalent to the family of SMP [3]. Thus, the SMP records the state of the process at each time point t, while the MRP is a point (counting) process which records the number of times each of the possible states has been visited up to time t. Note that the superposed arrival process is autocorrelated and is therefore not renewal.

In general, the task of optimal CAC for a link offered renewal per-category arrivals and exponential service times can be formulated as a semi-Markov decision process (SMDP) [16, 8]. In case of a Poisson arrival process, and exponential service process, the state vector represents the number of active calls from each category. The state transition probabilities, which are part of the MDP model, become easy to formulate. This is due to the memoryless property of the exponential distribution: the probability of the next event being an arrival/departure is independent of the time offset between the latest arrival and the latest departure. This is not the case if we replace the Poisson process with a non-Poisson process such as the Weibull process: the probability of the next event being an arrival/departure now becomes dependent on the time between the latest arrival and the latest departure.

Optimal CAC for one call category with renewal arrival process and Markov service process can be modeled in at least two ways. In the first way, which was described in a previous paper [12], the SMDP for the superposition of arrival and service process is constructed using the method of supplementary variables [2] The method of supplementary variables applies to the $GI/M/C$ and $GI/GI/C$ loss/queuing systems and the $GI/G/1$ queuing system. By introducing new state variables, which contain information about the type of the latest event, and the time offset between the latest event and the latest complementary event, the Markov property can be preserved, resulting in a SMDP. The state sojourn time of this SMDP will be non-exponential. The supplemenetary variables makes the state space very large. The second way, based on the embedded Markov chain method [11], was first proposed by Yechiali for access control of the $GI/M/C$ queue [18]. In this method the arrival instants are used as regeneration epochs of the Markov chain. Its main advantage is that the state space is sigificantly smaller than for the method of supplementary variables. The state sojourn time of the corresponding SMDP will be non-exponential. The basic method of the embedded controlled Markov chain is restricted to the one category case.

The case with multiple categories can be treated in at least two ways. Both ways involves the basic method of supplementary variables, either on its own, or in combination (hybrid) with the embedded controlled Markov chain method. The size of the state space will be smaller for the hybrid method which we recommend to use in case of two categories. In case of three or more categories the size of the state space will be very large even for the hybrid method.

For a full version of this paper, see [13], which presents up to date versions of the extended link MDP models, and a discussion on how to deal with the network case.

The paper is organized as follows. Section 2 formulates the CAC problem in terms of offered traffic and optimization objective. Section 3 describes different models of the call arrival process and service process. Section 4 outlines the MDP model for the embedded controlled Markov chain for a link operating in loss mode. Section 5 evaluates the standard and the link MDP models based on the method of supplementary variables and the method of embedded controlled Markov chain using numerical/simulation techniques. Finally, Section 6 concludes the paper.

2 Problem Formulation

We consider a single communication link with capacity C Mbps. The link is offered traffic from G categories which are, for sake of simplicity, assumed to be subject to deterministic multiplexing. The i-th category, $i \in I = \{1, \cdots, G\}$, is characterized by the following:

- Bandwidth requirement b_i [Mbps],
- General call arrival process $\{A_k\}$ with two special cases:
  - Poisson process with mean arrival rate λ_i [s^{-1}],
  - Weibull process characterized by scale parameter a_i and shape parameter c_i,
- Exponential service process $\{B_k\}$ with mean $1/\mu_i$ [s],
- Reward parameter $r_i \in (0, \infty)$

The task is to find an optimal link CAC policy π^* which maximizes the mean reward from the link, defined as

$$\overline{R}(\pi) = \sum_{i \in I} r_i \overline{\lambda}_i \tag{1}$$

where $\overline{\lambda}_i$ denotes the average category-i acceptance rate.

3 Modeling of Call Traffic

3.1 Arrival Process Model

Since the days of Erlang the Poisson model has commonly been used to describe the random arrivals of call requests to the OD pairs of a telephone network. Although the Poisson model serves its purpose in telephone networks, it lacks descriptive power in the case of Internet where a substantial portion of traffic is WWW, NNTP and SMTP connections transported by TCP. The WWW, NNTP and SMTP services produce connection arrivals which are different in nature from the telephone service; For example, a person using the WWW service is more likely to initiate additional downloads after the first download. A person using the telephone service is more likely to initiate independent calls.

Measurements on real WWW, NNTP and SMTP connection arrivals in the Internet have revealed that the arrival process shows burstiness over many time scales, ranging from seconds to hours. Paxson and Floyd found that actual

WAN traffic is consistent with statistical self-similarity for sufficiently large time scales [14]. These findings have been verified by Feldmann *et al.* [10].

Crovella has proposed an ON/OFF model for the downloading of web documents [4]. A single TCP connection is invoked in each ON period. The duration of the TCP connection follows a heavy-tailed distribution since the distribution of WWW document sizes on Internet is heavy-tailed. The OFF period corresponds to the user thinking time. Crovella argues that also the OFF period is heavy-tailed.

Deng also developed an ON/OFF model for the web service [5]. During the ON period, the user makes multiple web requests each resulting in a new TCP connection. The OFF period is the time between two ON periods while the user views the page. Deng proposed distributions for three parameters: the duration of the ON period was found to be Pareto distributed, the duration of the OFF period was found to be Weibull distributed as well as the inter-arrival time of web requests during the ON period.

Feldmann has proposed to model the arrivals of TCP connections by a Weibull interarrival time distribution [9]: $A(t) = P(A_k \leq t) = 1 - e^{-\left(\frac{t}{a}\right)^c}$, where k represent the discrete time index. The Weibull model provides an accurate fit to real TCP connection arrival statistics. The Weibull distribution has finite moments, including finite mean and variance of the inter-arrival time. For this reason the Weibull distribution is considered to have a light tail. Since its variance is finite the Weibull distribution does not give rise to a self-similar arrival process. Indeed, Downey claims [6] that there is little evidence that the times between WWW requests form a heavy-tailed distribution, which would give rise to the simplest form of self-similar traffic.

3.2 Service Process Model

The traditional model of call holding times B_k is the (negative) exponential distribution with rate parameter μ: $B(t) = \mathrm{P}(B_k \leq t) = 1 - \exp(-\mu t)$. The exponential distribution match the actual holding times in case of telephony among other services. However, for the WWW and FTP service, the connection holding time is more heavy tailed [4, 5, 14].

4 Link Model based on Embedded Markov Chain

A one-dimensional embedded Markov chain can be defined at the arrival instants. Let X_k be the number of calls found upon arrival of the kth call. Then X_k is a one-dimensional embedded Markov chain with state space

$$X = \{x : x = 0, 1, \ldots, \lfloor C/b \rfloor\}, \tag{2}$$

The control action space is given by

$$A = \{\theta \in \{0,1\}\}\,, \tag{3}$$

where $\theta = 0$ denotes call rejection and $\theta = 1$ denotes call acceptance. The permissible action space is a state-dependent subset of A:

$$A(x) = \{\theta \in A : \theta = 0 \text{ if } x+1 > \lfloor C/b \rfloor\}\,. \tag{4}$$

Let v_k be a stochastic process of discrete time $k = 1,2,\ldots$ representing the number of calls served between the arrival of the $(k-1)th$ and the kth call. Let θ_k be the admission decision for the kth call request. Then

$$X_{k+1} = X_k + \theta_k - v_{k+1}, \tag{5}$$

The state transition probabilities are:

$$p_{xy}(\theta) = \int_0^\infty \binom{x+\theta}{y} [1-e^{-\mu t}]^{x+\theta-y} e^{-y\mu t} dA(t). \tag{6}$$

The expected reward in state x is given by

$$R(x,\theta) = \sum_{y \in X} p_{xy}(\theta) r(x-y+\theta) \tag{7}$$

The mean sojourn time in state x is:

$$\tau(x,\theta) = \sum_{y \in X} \int_0^\infty t \binom{x+\theta}{y} [1-e^{-\mu t}]^{x+\theta-y} e^{-y\mu t} dA(t) = E[A_k] \tag{8}$$

5 Numerical Results

5.1 Considered Link Models

The performance analysis is performed for the single link case. Three MDP models for CAC are compared numerically:

- MDP – standard link model assuming Poisson call arrivals [8],
- MDP_S – extended link model based on the method of supplementary variables proposed in [12],
- MDP_E – extended link model based on the embedded Markov chain described in Section 4.

The integrals present in the MDP_S and MDP_E link models are solved by the Simpson's numerical integration method [15].

5.2 Examples and Results

The simulation scenario is described in Table 1. The traffic parameters are chosen such that the link load becomes moderate (83 % of link capacity). Each measurement period is based on 1×10^6 call events.

Table 1. Description of simulation scenario

link capacity C [Mbps]	24
traffic categories G	1
mean arrival rate λ [s^{-1}]	20
mean holding time $1/\mu$ [s]	1
bandwidth b [Mbps]	1
link traffic [Mbps*Erlang]	20
reward parameter r	1
#offset values N_ω	30-120

First, the Weibull shape parameter c is set to some value in the set $\{0.1, 0.2, \ldots, 1.0\}$. Second, the Weibull scale parameter a is set to $a = \frac{1}{\Gamma(1+\frac{1}{c})\lambda}$ which gives equal mean interarrival time for the Poisson and Weibull process.

The performance of CAC is evaluated in terms of the reward loss L:

$$L = 1 - \overline{R}/R, \tag{9}$$

where $\overline{R} = \sum_{i \in I} r_i \overline{\lambda}_i$ and $R = \sum_{i \in I} r_i \lambda_i$ denotes the carried and offered reward rate, respectively.

The optimal access policy for a single call category is to always accept a new call when there is sufficient free capacity on the link. The corresponding access policy is known as complete sharing (CS). The loss results for MDP, MDP_S and MDP_E are obtained by solving a linear equation system for the relative values and the average reward rate $\overline{R}$. We evaluate the modeling accuracy of the MDP_S and MDP_E methods by the average reward rate. We expect that the accuracy of the relative values will be similar to the accuracy of the average reward rate.

5.3 Results Analysis

From the graphs in Figure 1 the following conclusions are drawn:

– The standard MDP method models the reward loss accurately only for Poisson traffic (c=1),
– The MDP_S method estimates the reward loss accurately.

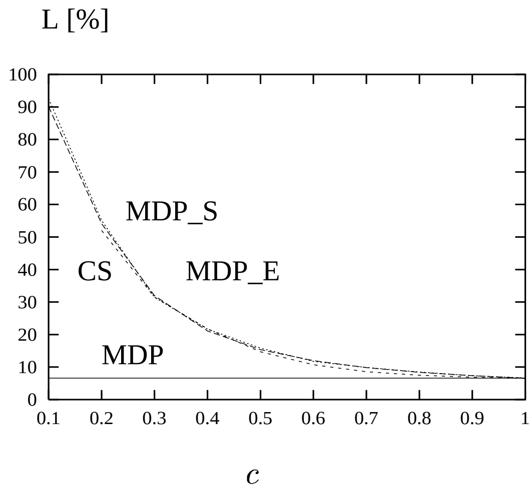

Fig. 1. Reward loss versus Weibull shape parameter c.

– The MDP_E method estimates the reward loss accurately.

The number of offset values, N_ω, was adapted to the Weibull parameter c. A smaller c parameter required a larger N_ω value. The case $c = 0.1$ was not considered by the MDP_S method due to the large state space associated with the large N_ω value.

6 Conclusion

In this paper we formulated the CAC problem for a single link operating in loss mode. In this formulation each call category is characterized by its reward parameter defining the expected reward for carrying a call from this category. Such a formulation allows to apply Markov Decision Process (MDP) theory to solve the problem.

Traditionally, the MDP approach to CAC and routing has assumed Poisson call arrivals and exponentially distributed call holding times. These assumptions are reasonable for telephone calls. However, they become inaccurate for the TCP connections invoked within the WWW, NNTP and SMTP Internet services. In particular, measurements on real Internet traffic have revealed that the TCP connection arrival process is self-similar and that TCP connection holding time distribution is more heavy tailed than the standard exponential distribution.

In this paper we adopt a simple model of call traffic since this simplifies the MDP model. Call arrivals are modeled by a non-self-similar renewal model with Weibull distributed interarrival times [9]. Call service times are modeled by an exponential distribution. Given this traffic model, we propose an extended MDP model for a single call category. The MDP model is based on the embedded Markov chain method [11]. The numerical results show that the proposed link MDP model provides accurate estimates of the reward loss.

A long-term goal of this work is near-optimal CAC and routing on the network level, with general arrivals from multiple call categories. In order to meet these goals we need to design an extended link MDP for general arrivals from multiple call categories, as well as a model of the superposed arrival process to a network link.

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